NAG Toolbox for MATLAB

s14ae

1 Purpose

s14ae returns the value of the kth derivative of the psi function $\psi(x)$ for real x and $k = 0, 1, \ldots, 6$, via the function name.

2 Syntax

```
[result, ifail] = s14ae(x, k)
```

3 Description

s14ae evaluates an approximation to the kth derivative of the psi function $\psi(x)$ given by

$$\psi^{(k)}(x) = \frac{d^k}{dx^k}\psi(x) = \frac{d^k}{dx^k}\left(\frac{d}{dx}\log_e \Gamma(x)\right),$$

where x is real with $x \neq 0, -1, -2, \ldots$ and $k = 0, 1, \ldots, 6$. For negative noninteger values of x, the recurrence relationship

$$\psi^{(k)}(x+1) = \psi^{(k)}(x) + \frac{d^k}{dx^k}\left(\frac{1}{x}\right)$$

is used. The value of $\frac{(-1)^{k+1}\psi^{(k)}(x)}{k!}$ is obtained by a call to s14ad, which is based on the function PSIFN in Amos 1983.

Note that $\psi^{(k)}(x)$ is also known as the *polygamma* function. Specifically, $\psi^{(0)}(x)$ is often referred to as the *digamma* function and $\psi^{(1)}(x)$ as the *trigamma* function in the literature. Further details can be found in Abramowitz and Stegun 1972.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Amos D E 1983 Algorithm 610: A portable FORTRAN subroutine for derivatives of the psi function *ACM Trans. Math. Software* **9** 494–502

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

Constraint: **x** must not be 'too close' (see Section 6) to a nonpositive integer.

2: **k – int32 scalar**

The function $\psi^{(k)}(x)$ to be evaluated.

Constraint: $0 \leq \mathbf{k} \leq 6$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{k} < 0$,
or $\mathbf{k} > 6$,
or $\mathbf{x}$ is 'too close' to a nonpositive integer. That is, $\mathrm{ABS}(\mathbf{x} - \mathrm{NINT}(\mathbf{x})) <$ ***machine precision*** $\times\ \mathrm{NINT}(\mathrm{ABS}(\mathbf{x}))$.

ifail $= 2$

The evaluation has been abandoned due to the likelihood of underflow. The result is returned as zero.

ifail $= 3$

The evaluation has been abandoned due to the likelihood of overflow. The result is returned as zero.

7 Accuracy

All constants in s14ad are given to approximately 18 digits of precision. If t denotes the number of digits of precision in the floating-point arithmetic being used, then clearly the maximum number in the results obtained is limited by $p = \min(t, 18)$. Empirical tests by Amos 1983 have shown that the maximum relative error is a loss of approximately two decimal places of precision. Further tests with the function $-\psi^{(0)}(x)$ have shown somewhat improved accuracy, except at points near the positive zero of $\psi^{(0)}(x)$ at $x = 1.46\ldots$, where only absolute accuracy can be obtained.

8 Further Comments

None.

9 Example

```
x = 2.5;
k = int32(2);
[result, ifail] = s14ae(x, k)
```

```
result =
```

```
   -0.2362
ifail =
              0
```